Moment Method

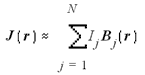
The method of moments (MoM) technique is based upon the work of R.F. Harrington, an electrical engineer who worked extensively on the method and applied it to electromagnetic field problems, in the beginning of the 1960's. It is based on older theory which uses weighted residuals and variational calculus. More detailed information on the method of moments and Green's theorem can be found in [*Field Computation by Moment Methods* (1)](http://literature.cdn.keysight.com/litweb/pdf/ads2008/mom/ads2008/Theory_of_Operation_for_Momentum.html#TheoryofOperationforMomentum-1108315).  
In the method of moments, prior to the discretization, Maxwell's electromagnetic equations are transformed into integral equations. These follow from the definition of suitable electric and magnetic Green's functions in the multilayered substrate.

In Momentum, a mixed potential integral equation (MPIE) formulation is used. This formulation expresses the electric and magnetic field as a combination of a vector and a scalar potential. The unknowns are the electric and magnetic surface currents flowing in the planar circuit.

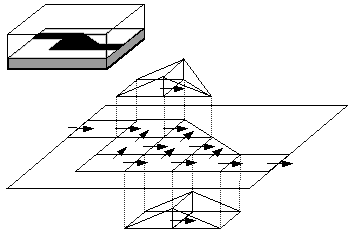
Using notations from linear algebra, we can write the mixed potential integral equation in very general form as a linear integral operator equation:

http://literature.cdn.keysight.com/litweb/pdf/ads2008/mom/3125407/mom-15-1-01.gif (1)

Here, J(r) represents the unknown surface currents and E(r) the known excitation of the problem. The Green's dyadic of the layered medium acts as the integral kernel. The unknown surface currents are discretized by meshing the planar metalization patterns and applying an expansion in a finite number of subsectional basis functions B1(r), ..., BN(r):

 (2)

The standard basis functions used in planar EM simulators are the subsectional rooftop functions defined over the rectangular, triangular, and polygonal cells in the mesh. Each rooftop is associated with one edge of the mesh and represents a current with constant density flowing through that edge as shown in the following illustration. The unknown amplitudes Ij, j=1,..,N of the basis function expansion determine the currents flowing through all edges of the mesh.



**Discretization of the surface currents using rooftop basis functions.**

The integral equation [(1)](http://literature.cdn.keysight.com/litweb/pdf/ads2008/mom/ads2008/Theory_of_Operation_for_Momentum.html#TheoryofOperationforMomentum-eqa1) is discretized by inserting the rooftop expansion [(2)](http://literature.cdn.keysight.com/litweb/pdf/ads2008/mom/ads2008/Theory_of_Operation_for_Momentum.html#TheoryofOperationforMomentum-eqa2) of the currents. By applying the Galerkin testing procedure, that is, by testing the integral equation using test functions identical to the basis functions, the continuous integral equation [(1)](http://literature.cdn.keysight.com/litweb/pdf/ads2008/mom/ads2008/Theory_of_Operation_for_Momentum.html#TheoryofOperationforMomentum-eqa1) is transformed into a discrete matrix equation:

for i=1,...,N

http://literature.cdn.keysight.com/litweb/pdf/ads2008/mom/3125407/mom-15-1-04.gif (3)

with

http://literature.cdn.keysight.com/litweb/pdf/ads2008/mom/3125407/mom-15-1-05.gif (4)

http://literature.cdn.keysight.com/litweb/pdf/ads2008/mom/3125407/mom-15-1-06.gif (5)

The left hand side matrix [Z] is called the interaction matrix, as each element in this matrix describes the electromagnetic interaction between two rooftop basis functions. The dimension N of [Z] is equal to the number of basis functions. The right-hand side vector [V] represents the discretized contribution of the excitations applied at the ports of the circuit.

The surface currents contribute to the electromagnetic field in the circuit by means of the Green's dyadic of the layer stack. In the MPIE formulation, this Green's dyadic is decomposed into a contribution from the vector potential A(r) and a contribution from the scalar potential V(r):

http://literature.cdn.keysight.com/litweb/pdf/ads2008/mom/3125407/mom-15-1-07.gif (6)

The scalar potential originates from the dynamic surface charge distribution derived from the surface currents and is related to the vector potential through the Lorentz gauge.  
By substituting the expression [(6)](http://literature.cdn.keysight.com/litweb/pdf/ads2008/mom/ads2008/Theory_of_Operation_for_Momentum.html#TheoryofOperationforMomentum-eqa6) for the Green's dyadic in the expression [(4)](http://literature.cdn.keysight.com/litweb/pdf/ads2008/mom/ads2008/Theory_of_Operation_for_Momentum.html#TheoryofOperationforMomentum-eqa4) for the interaction matrix elements, yields the following form:

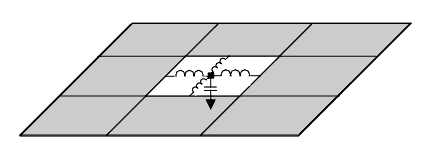
http://literature.cdn.keysight.com/litweb/pdf/ads2008/mom/3125407/mom-15-1-08.gif (7)

with

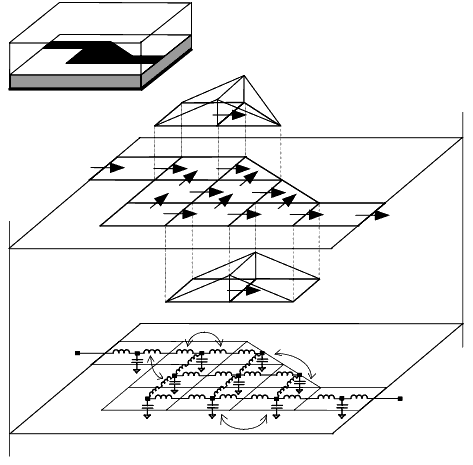
http://literature.cdn.keysight.com/litweb/pdf/ads2008/mom/3125407/mom-15-1-09.gif (8)

http://literature.cdn.keysight.com/litweb/pdf/ads2008/mom/3125407/mom-15-1-10.gif (9)

This allows the interaction matrix equation to be given a physical interpretation by constructing an equivalent network model see [Figure A-2](http://literature.cdn.keysight.com/litweb/pdf/ads2008/mom/ads2008/Theory_of_Operation_for_Momentum.html#TheoryofOperationforMomentum-1104441). In this network, the nodes correspond to the cells in the mesh and hold the cell charges. Each cell corresponds to a capacitor to the ground. All nodes are connected with branches which carry the current flowing through the edges of the cells. Each branch has in inductor representing the magnetic self coupling of the associated current basis function. All capacitors and inductors in the network are complex, frequency dependent and mutually coupled, as all basis functions interact electrically and magnetically see [Figure A-3](http://literature.cdn.keysight.com/litweb/pdf/ads2008/mom/ads2008/Theory_of_Operation_for_Momentum.html#TheoryofOperationforMomentum-1104447). The ground in this equivalent network corresponds with the potential at the infinite metallization layers taken up in the layer stack. In the absence of infinite metallization layers, the ground corresponds with the sphere at infinity. The method of moments interaction matrix equation follows from applying the Kirchoff voltage laws in the equivalent network. The currents in the network follow from the solution of the matrix equation and represent the amplitudes of the basis functions.

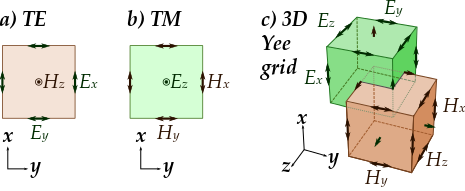


**Figure A-2. The equivalent circuit is built by replacing each cell in the mesh with a capacitor to the ground reference and inductors to the neighboring cells.**



**Figure A-3. Equivalent network representation of the discretized MoM problem.**

FTTD



**Finite-difference time-domain** or **Yee's method** (named after the Chinese American applied mathematician [Kane S. Yee](https://en.wikipedia.org/w/index.php?title=Kane_S._Yee&action=edit&redlink=1), born 1934) is a [numerical analysis](https://en.wikipedia.org/wiki/Numerical_analysis) technique used for modeling [computational electrodynamics](https://en.wikipedia.org/wiki/Computational_electrodynamics) (finding approximate solutions to the associated system of [differential equations](https://en.wikipedia.org/wiki/Differential_equation)). Since it is a [time-domain](https://en.wikipedia.org/wiki/Time_domain) method, FDTD solutions can cover a wide [frequency](https://en.wikipedia.org/wiki/Frequency) range with a single [simulation](https://en.wikipedia.org/wiki/Computer_simulation) run, and treat nonlinear material properties in a natural way.

The FDTD method belongs in the general class of [grid](https://en.wikipedia.org/wiki/Discretization)-based differential numerical modeling methods ([finite difference methods](https://en.wikipedia.org/wiki/Finite_difference_methods)). The time-dependent [Maxwell's equations](https://en.wikipedia.org/wiki/Maxwell%27s_equations) (in [partial differential](https://en.wikipedia.org/wiki/Partial_differential_equation) form) are discretized using [central-difference](https://en.wikipedia.org/wiki/Central_difference) approximations to the space and time [partial derivatives](https://en.wikipedia.org/wiki/Partial_derivative). The resulting [finite-difference](https://en.wikipedia.org/wiki/Finite_difference_method) equations are solved in either software or hardware in a [leapfrog](https://en.wikipedia.org/wiki/Leapfrog_integration) manner: the [electric field](https://en.wikipedia.org/wiki/Electric_field) [vector components](https://en.wikipedia.org/wiki/Vector_component) in a volume of space are solved at a given instant in time; then the [magnetic field](https://en.wikipedia.org/wiki/Magnetic_field) vector components in the same spatial volume are solved at the next instant in time; and the process is repeated over and over again until the desired transient or steady-state electromagnetic field behavior is fully evolved.

When [Maxwell's differential equations](https://en.wikipedia.org/wiki/Maxwell%27s_differential_equations) are examined, it can be seen that the change in the E-field in time (the time derivative) is dependent on the change in the H-field across space (the [curl](https://en.wikipedia.org/wiki/Curl_(mathematics))). This results in the basic FDTD time-stepping relation that, at any point in space, the updated value of the E-field in time is dependent on the stored value of the E-field and the numerical curl of the local distribution of the H-field in space.[[2]](https://en.wikipedia.org/wiki/Finite-difference_time-domain_method#cite_note-yee66-2)

The H-field is time-stepped in a similar manner. At any point in space, the updated value of the H-field in time is dependent on the stored value of the H-field and the numerical curl of the local distribution of the E-field in space. Iterating the E-field and H-field updates results in a marching-in-time process wherein sampled-data analogs of the continuous electromagnetic waves under consideration propagate in a numerical grid stored in the computer memory.

This description holds true for 1-D, 2-D, and 3-D FDTD techniques. When multiple dimensions are considered, calculating the numerical curl can become complicated. Kane Yee's seminal 1966 paper proposed spatially staggering the vector components of the E-field and H-field about rectangular unit cells of a Cartesian computational grid so that each E-field vector component is located midway between a pair of H-field vector components, and conversely.[[2]](https://en.wikipedia.org/wiki/Finite-difference_time-domain_method#cite_note-yee66-2) This scheme, now known as a **Yee lattice**, has proven to be very robust, and remains at the core of many current FDTD software constructs.

Furthermore, Yee proposed a leapfrog scheme for marching in time wherein the E-field and H-field updates are staggered so that E-field updates are conducted midway during each time-step between successive H-field updates, and conversely.[[2]](https://en.wikipedia.org/wiki/Finite-difference_time-domain_method#cite_note-yee66-2) On the plus side, this explicit time-stepping scheme avoids the need to solve simultaneous equations, and furthermore yields dissipation-free numerical wave propagation. On the minus side, this scheme mandates an upper bound on the time-step to ensure numerical stability.[[9]](https://en.wikipedia.org/wiki/Finite-difference_time-domain_method#cite_note-taflove75a-9) As a result, certain classes of simulations can require many thousands of time-steps for completion.

### Strengths of FDTD modeling**[[edit](https://en.wikipedia.org/w/index.php?title=Finite-difference_time-domain_method&action=edit&section=5" \o "Edit section: Strengths of FDTD modeling)]**

Every modeling technique has strengths and weaknesses, and the FDTD method is no different.

* FDTD is a versatile modeling technique used to solve Maxwell's equations. It is intuitive, so users can easily understand how to use it and know what to expect from a given model.
* FDTD is a time-domain technique, and when a broadband pulse (such as a Gaussian pulse) is used as the source, then the response of the system over a wide range of frequencies can be obtained with a single simulation. This is useful in applications where resonant frequencies are not exactly known, or anytime that a broadband result is desired.
* Since FDTD calculates the E and H fields everywhere in the computational domain as they evolve in time, it lends itself to providing animated displays of the electromagnetic field movement through the model. This type of display is useful in understanding what is going on in the model, and to help ensure that the model is working correctly.
* The FDTD technique allows the user to specify the material at all points within the computational domain. A wide variety of linear and nonlinear dielectric and magnetic materials can be naturally and easily modeled.
* FDTD allows the effects of apertures to be determined directly. Shielding effects can be found, and the fields both inside and outside a structure can be found directly or indirectly.
* FDTD uses the E and H fields directly. Since most EMI/EMC modeling applications are interested in the E and H fields, it is convenient that no conversions must be made after the simulation has run to get these values.

### Weaknesses of FDTD modeling**[[edit](https://en.wikipedia.org/w/index.php?title=Finite-difference_time-domain_method&action=edit&section=6" \o "Edit section: Weaknesses of FDTD modeling)]**

* Since FDTD requires that the entire computational domain be gridded, and the grid spatial discretization must be sufficiently fine to resolve both the smallest electromagnetic wavelength and the smallest geometrical feature in the model, very large computational domains can be developed, which results in very long solution times. Models with long, thin features, (like wires) are difficult to model in FDTD because of the excessively large computational domain required. Methods such as [Eigenmode Expansion](https://en.wikipedia.org/wiki/Eigenmode_Expansion" \o "Eigenmode Expansion) can offer a more efficient alternative as they do not require a fine grid along the z-direction.[[73]](https://en.wikipedia.org/wiki/Finite-difference_time-domain_method#cite_note-phot_cad-73)
* There is no way to determine unique values for permittivity and permeability at a material interface.
* Space and time steps must satisfy the [CFL condition](https://en.wikipedia.org/wiki/Courant%E2%80%93Friedrichs%E2%80%93Lewy_condition), or the [leapfrog integration](https://en.wikipedia.org/wiki/Leapfrog_integration) used to solve the partial differential equation is likely to become unstable.
* FDTD finds the E/H fields directly everywhere in the computational domain. If the field values at some distance are desired, it is likely that this distance will force the computational domain to be excessively large. Far-field extensions are available for FDTD, but require some amount of postprocessing.[[4]](https://en.wikipedia.org/wiki/Finite-difference_time-domain_method#cite_note-taflove05-4)
* Since FDTD simulations calculate the E and H fields at all points within the computational domain, the computational domain must be finite to permit its residence in the computer memory. In many cases this is achieved by inserting artificial boundaries into the simulation space. Care must be taken to minimize errors introduced by such boundaries. There are a number of available highly effective absorbing boundary conditions (ABCs) to simulate an infinite unbounded computational domain.[[4]](https://en.wikipedia.org/wiki/Finite-difference_time-domain_method#cite_note-taflove05-4) Most modern FDTD implementations instead use a special absorbing "material", called a [perfectly matched layer](https://en.wikipedia.org/wiki/Perfectly_matched_layer) (PML) to implement absorbing boundaries.[[42]](https://en.wikipedia.org/wiki/Finite-difference_time-domain_method#cite_note-berenger94-42)[[46]](https://en.wikipedia.org/wiki/Finite-difference_time-domain_method#cite_note-gedney96-46)
* Because FDTD is solved by propagating the fields forward in the time domain, the electromagnetic time response of the medium must be modeled explicitly. For an arbitrary response, this involves a computationally expensive time convolution, although in most cases the time response of the medium (or [Dispersion (optics)](https://en.wikipedia.org/wiki/Dispersion_(optics))) can be adequately and simply modeled using either the recursive convolution (RC) technique, the auxiliary differential equation (ADE) technique, or the Z-transform technique. An alternative way of solving [Maxwell's equations](https://en.wikipedia.org/wiki/Maxwell%27s_equations) that can treat arbitrary dispersion easily is the [Pseudospectral Spatial-Domain method (PSSD)](https://en.wikipedia.org/wiki/Computational_electrodynamics" \l "Pseudo-spectral_spatial_domain_.28PSSD.29" \o "Computational electrodynamics), which instead propagates the fields forward in space.

### Grid truncation techniques

The most commonly used grid truncation techniques for open-region FDTD modeling problems are the Mur absorbing boundary condition (ABC),[[13]](https://en.wikipedia.org/wiki/Finite-difference_time-domain_method#cite_note-mur81-13) the Liao ABC,[[16]](https://en.wikipedia.org/wiki/Finite-difference_time-domain_method#cite_note-liao84-16) and various [perfectly matched layer](https://en.wikipedia.org/wiki/Perfectly_matched_layer) (PML) formulations.[[4]](https://en.wikipedia.org/wiki/Finite-difference_time-domain_method#cite_note-taflove05-4)[[42]](https://en.wikipedia.org/wiki/Finite-difference_time-domain_method#cite_note-berenger94-42)[[46]](https://en.wikipedia.org/wiki/Finite-difference_time-domain_method#cite_note-gedney96-46) The Mur and Liao techniques are simpler than PML. However, PML (which is technically an absorbing region rather than a boundary condition *per se*) can provide orders-of-magnitude lower reflections. The PML concept was introduced by J.-P. Berenger in a seminal 1994 paper in the Journal of Computational Physics.[[42]](https://en.wikipedia.org/wiki/Finite-difference_time-domain_method#cite_note-berenger94-42) Since 1994, Berenger's original split-field implementation has been modified and extended to the uniaxial PML (UPML), the convolutional PML (CPML), and the higher-order PML. The latter two PML formulations have increased ability to absorb evanescent waves, and therefore can in principle be placed closer to a simulated scattering or radiating structure than Berenger's original formulation.

To reduce undesired numerical reflection from the PML additional back absorbing layers technique can be used.[[74]](https://en.wikipedia.org/wiki/Finite-difference_time-domain_method#cite_note-back_pml-74)